Logarithmic Functions and Applications

Math 130 - Essentials of Calculus

17 September 2019

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4. *Solve for x in the equation:* $e^{-x} = 5$

Applications

Example

A small-appliance manufacturer finds that it costs \$9000 *to produce* 1000 *toaster ovens a week and* \$12,000 *to produce* 1500 *a week.*

1. *Express the cost as a function of the number of toaster ovens produced, assuming that it is linear.*
2. *What is the slope of the graph and what does is represent?*
3. *What is the* y*-intercept of the graph and what does it represent?*

Applications

Example

Environmental scientists measure the intensity of light at various depths in a lake to find the "transparency" of the water. Certain levels of transparency are required for the biodiversity of the submerged macrophyte population. In a certain lake the intensity of light at a depth of x feet is given by

$$I = 10e^{-0.008x}$$

where I is measured in lumens. At what depth has the light intensity dropped to 5 lumens?

Applications

Example

Suppose you're driving a car on a cold winter day ($20^\circ F$ outside) and the engine overheats (at about $220^\circ F$). When you park, the engine begins to cool down. The temperature T of the engine x minutes after you park satisfies the equation

$$\ln\left(\frac{T-20}{200}\right) = -0.11x$$

Find the temperature of the engine after 20 minutes.

Applications

Example

Suppose the function $P(t) = 437.2(1.036)^t$ *is used to model the population, measured in thousands of people, of a country* t *years after the end of* 1995*. When will the population reach one million people?*

Applications

Example

Suppose you are offered a job that lasts one month. Which of the following methods of payment do you prefer?

1. *One million dollars at the end of the month.*
2. *One cent on the first day of the month, two cents on the second day, four cents on the third day, and, in general,* 2^{n-1} *cents on the nth day.*

Applications

Example

People are moving into a small community and driving the home prices higher. Suppose $p(t)$ is the population of the community t years after January 1, 2000, and $f(n)$ is the median home price when the population of the area is n people. Which function gives a meaningful result, $p(f(n))$ or $f(p(t))$? What does it represent in the context?